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**Chapter 11**

WAVEGUIDES

**11.1 Introduction**

In the previous chapters, a pair of conductors was used to guide electromagnetic wave propagation. This propagation was via the transverse electromagnetic (TEM) mode, meaning both the electric and magnetic field components were transverse, or perpendicular, to the direction of propagation. In this chapter we investigate wave-guiding structures that support propagation in non-TEM modes, namely in the transverse electric (TE) and transverse magnetic (TM) modes.

A waveguide guides or directs electromagnetic waves from one region of space to another. There are two primary purposes of waveguides: (1) Guide power from one point to another (i.e. a power line) and (2) Transmit information from one point to another. A hollow metallic tube of uniform cross section for transmitting electromagnetic waves by successive reflections from the inner walls of the tube is called a waveguide. At frequencies higher than 3GHz transmission of electromagnetic waves along transmission lines and coaxial cables becomes difficult due to skin effect, dielectric losses. A metallic tube can be used to transmit electromagnetic wave at these frequencies.

Waveguides can be constructed to carry waves over a wide portion of the [electromagnetic spectrum](http://en.wikipedia.org/wiki/Electromagnetic_spectrum), but are especially useful in the [microwave](http://en.wikipedia.org/wiki/Microwave) and [optical](http://en.wikipedia.org/wiki/Optics) frequency ranges. Depending on the frequency, they can be constructed from either [conductive](http://en.wikipedia.org/wiki/Electrical_conduction) or [dielectric](http://en.wikipedia.org/wiki/Dielectric) materials. Waveguides are used for transferring both [power](http://en.wikipedia.org/wiki/Power_(physics)) and communication signals. Generally waveguides are constructed of brass, copper or aluminum. The inner surface of the waveguide is usually coated with either gold or silver to improve the conductivity and minimize the losses inside the waveguide. In waveguides the electric and magnetic fields are confined to the space within the guides. Thus no power is lost to radiation. Since the guides are normally filled with air, dielectric losses are negligible. However, there is some I2R power lost to heat in the walls of the guides, but this loss is usually very small.

In fact any configuration of electric and magnetic fields inside a waveguide must solution of Maxwell’s equations. These fields must also satisfy the boundary conditions imposed by the walls of the guide. It is of utmost importance to realize that conduction of energy takes place not through the walls of the guide; the guide walls only confine the energy. Actual energy conduction takes place through the dielectric filling the waveguide, usually air.

It is possible to propagate several modes of electromagnetic waves within a waveguide. The physical dimensions of a waveguide determine the cutoff frequency for each mode. If the frequency of the impressed signal is above the cutoff frequency for a given mode, the electromagnetic energy can be transmitted through the guide for that particular mode with minimal attenuation. Otherwise the electromagnetic energy with a frequency below cutoff for that particular mode will be attenuated to a negligible value in a relatively short distance. The dominant mode in a particular guide is the mode having the lowest cut off frequency

In wave propagation it is necessary to speak in terms of electric and magnetic fields while discussing the properties of waveguide, unlike in transmission lines when we speak in terms of voltage and current. Waveguides have certain advantages comparison to coaxial lines these are

1. Higher power handling capability
2. Lower loss per unit length
3. A simpler, low cost structure
4. The large surface area of waveguides greatly reduces copper (I2R) losses.

Waveguides, like transmission lines, are structures used to guide electromagnetic waves from point to point. However, the fundamental characteristics of waveguide and transmission line waves (*modes*) are quite different. The differences in these modes result from the basic differences in geometry for a transmission line and a waveguide. Comparison of waveguide and Transmission line as given in Table 11.1

**Table 11.1 Comparison of Waveguide and Transmission Line Characteristics**

|  |  |
| --- | --- |
| Transmission line | Waveguide |
| 1. Two or more conductors separated by some insulating medium (two-wire, coaxial, micro strip, etc.). | 1. Metal waveguides are typically one enclosed conductor filled with an insulating medium (rectangular, circular) while a dielectric waveguide consists of multiple dielectrics. |
| 1. Normal operating mode is the TEM or quasi-TEM mode (can support TE and TM modes but these modes are typically undesirable). | 1. Operating modes are TE or TM modes (cannot support a TEM mode). |
| 1. No cutoff frequency for the TEM mode. Transmission lines can transmit signals from DC up to high frequency. | 1. Must operate the waveguide at a frequency above the respective TE or TM mode cutoff frequency for that mode to propagate. |
| 1. Significant signal attenuation at high frequencies due to conductor and dielectric losses. | 1. Lower signal attenuation at high frequencies than transmission lines. |
| 1. Small cross-section transmission lines (like coaxial cables) can only transmit low power levels due to the relatively high fields concentrated at specific locations within the device. | 1. Metal waveguides can transmit high power levels. The fields of the propagating wave are spread more uniformly over a larger cross-sectional area than the small cross-section transmission line. |
| 1. Large cross-section transmission lines (like power transmission lines) can transmit high power levels. | 1. Large cross-section (low frequency) waveguides are impractical due to large size and high cost. |

**11.2 Types of wave guides**

A waveguide is a structure which guides waves, such as [electromagnetic waves](http://en.wikipedia.org/wiki/Electromagnetic_wave). There are different types of waveguides for each type of wave. Waveguide consists of a hollow metallic tube of arbitrary cross section uniform in extent in the direction of propagation. They are constructed from conductive material and may be rectangular, circular, or elliptical in shape. It can be rigid or flexible. Waveguides have very low loss. Any shape of cross section of a waveguide can support electromagnetic waves. But since irregular shapes are difficult to analyze and are rarely used, rectangular and circular waveguides have become more common. Waveguides of various shapes as shown in Figure 11.1

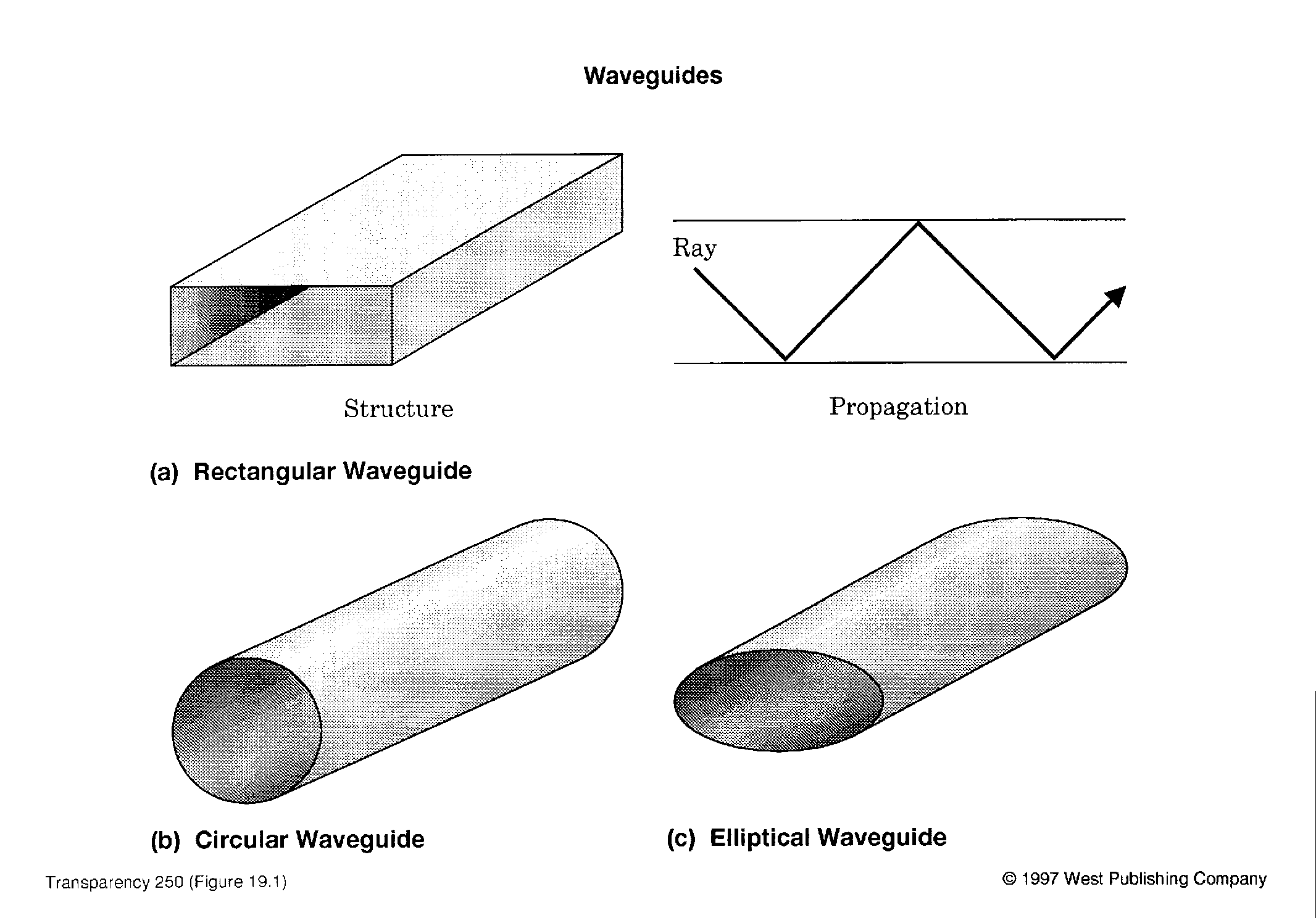


Figure11.1 Waveguides of various shapes (a) Rectangular (b) Circular (c) Elliptical



Rectangular waveguide (Fig 11.1a) is most common. Circular waveguide tends to twist the waves as these travels through them. However Circular waveguides (Fig 11.1b) are used with rotating antennas as in radars. Elliptical shape is often preferred in flexible waveguides (Fig 11.1c). Flexible waveguides will be required whenever the waveguide section should be capable of movement, like bending stretching or twisting. This must not cause deterioration in the performance of the waveguide. A copper tube having an elliptical cross section is a good example of a flexible waveguide.

Ridging is a convenient method of reducing the waveguide dimensions and thereby increase the critical wavelength but the presence of ridge has disadvantage of increased attenuation, reduced power handling capability and introducing distortions.

**11.3 Rectangular Waveguides**

Rectangular waveguides are the one of the earliest type of the transmission lines. They are ideally suited for high-power and low-loss microwave applications. A large variety of components such as couplers, detectors, isolators, attenuator, and slotted lines are commercially available for various standard waveguide bands from 1 GHz to over 220 GHz. Rectangular waveguide is a hollow metallic tube with a rectangular cross section. The conducting walls of the guide confine the electromagnetic fields and thereby guide the electromagnetic wave.

A plane wave in a waveguide resolves into two components (1) standing wave (2) traveling wave. Standing wave is in the direction normal to the reflecting walls of the waveguide. Traveling wave is in the direction parallel to the reflecting walls. The rectangular waveguide has a width “a” and height “b” as shown in figure 11.2. This waveguide is simply a brass or copper pipe having a rectangular cross section.

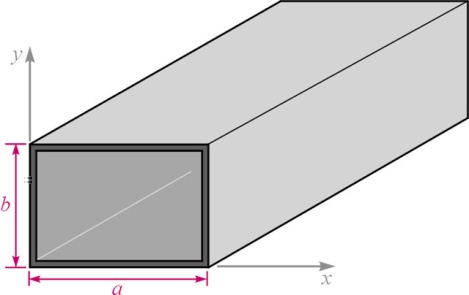


Figure 11.2 Rectangular waveguide

Rectangular waveguides are normally made in standard sizes with width “a” approximately twice the height “b”. The “a” dimension cannot be less than one-half wavelength. This can be seen since the guide is made up of two quarter-wavelength stubs separated by a small distance. Any frequency that makes the “a” dimension less than one-half wavelength allows no propagation of energy down the waveguide. The frequency that makes the “a” dimension look like exactly one-half wavelength is called the *cutoff frequency*, *fc*.

A rectangular waveguide cannot propagate below some certain frequency. This frequency is called the cut-off frequency.

The hollow rectangular waveguide can propagate TM and TE modes, but not TEM waves, since only one conductor is present. We will see that the TM and TE modes of a rectangular waveguide have cutoff frequencies below which propagation is not possible, similar to the TM and TE modes of the parallel plate guide. For waveguide, we either look at transverse electric (TE) or transverse magnetic (TM) waves. There is no combination in waveguide. These are the “modes” of operation for microwave energy in a waveguide structure.

In addition to the TE or TM designation, subscripts are used to describe the electric and magnetic field configuration. The general symbol is TEmn and TMmn where the subscript “m” indicates the number of half wave variations of the electric field along the “a” (wide) dimension of the guide and the subscript “n” is the number of half-wave variations of the electric field in the “b” (narrow) dimension. The most common mode of operation and the mode that has the longest operating wavelength is the TE10 mode. Simultaneous existence of two different modes with the same phase velocity is possible. Such modes are called degenerate modes.

Due to the non-TEM nature of the supported mode, the waveguide exhibits a high-pass filter-like behavior. Also due to the non uniqueness of the voltage and the current, the characteristic impedance, Z0, cannot be uniquely defined. Z0 may be defined in terms of the voltage-current ratio, the power-current ratio, or the power-voltage ratio.

**11.3.1 Field equations between rectangular wave guide**

Let us consider a rectangular waveguide situated in the rectangular coordinate system with its breadth along the x-axis width along y axis and wave propagates along the z direction, waveguide is filled with air as dielectric as shown in Figure 11.3.

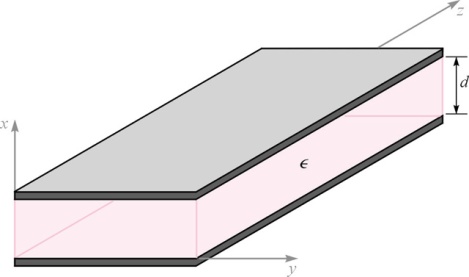


Figure 11.3 Propagation through a rectangular waveguide

We will assume that the wave guide is invariant in the z-direction and that the wave is propagating in z as (We could also have assumed propagation in z-direction.)

The electric and magnetic wave equations in frequency domain are given by



For TM wave (Hz=0) (11.1)



For TE wave (Ez=0) (11.2)

Expanding 𝛻2E in terms of rectangular coordinate system



(11.3)

Since the wave is propagating in the ‘z’ direction we have the operator



( ) (11.4)

Substituting equation 11.4 in equation 11.3 we get



(11.5)



(11.6)



Let be a constant then equation can be rewritten as



For TM waves (11.7)

Similarly, For TE waves (11.8)



By solving above partial differential equations, we get solutions of Ez and Hz. Using Maxwell’s equation, it is possible to find the various components along x and y directions [Ex, Hx, Ey, Hy].



Proceeding from the Maxwell curl equation ( (11.9)

Expanding the 1st Maxwell’s equation

=  =  (11.10)



Replacing (an operator), we get

 =  (11.11)

Equating coefficients of,  and  (after expanding) we get

 (11.12)

 (11.13)

 (11.14)



Proceeding from the Maxwell curl equation ( , B=)

(11.15)

  (11.16)



Replacing (an operator), we get

  (11.17)

Equating coefficients of,  and  (after expanding) we get

 (11.18)

 (11.19)

 (11.20)

From equation (11.12) to equation (11.14) and (11.18) to equation (11.20) are used to produce simple algebraic equations for the transverse components of E and H(x and y). From equation (11.19)

Using these equations, we can find expressions for the four transverse components [Ex , Hx, Ey, and Hy] in terms of the z-directed components (Ez and Hz). For instance, solving equation (11.20) for Hy we find

 (11.21)

Inserting this value of Hy into substituting equation (11.21) in equation

 (11.22)

 (11.23)

 (11.24)

 (11.25)

These equations give a general relationship for field components within a waveguide. The equations (11.22) to (11. 25) are solutions to Maxwell’s equations can be by substitutions. Note that the propagation coefficient γ is not known. So there are seven scalar unknowns (the six field components and γ)

We can see that all transverse components of E and H can be determined from only the axial components Ez and Hz It is this fact that allows the mode designations TEM, TE and TM. Furthermore, we can use superposition to reduce the complexity of the solution by considering each of these mode types, then adding the fields together at the end.

From equations (11.22), and (11.25), we notice that there are different types of field patterns or configurations. Each of these distinct field patterns is called a *mode.* Four different mode categories can exist, namely: 1. Hz = 0 and Ez = 0 (TEM mode), 2. Ez = 0 and Hz 0(TE modes), Ez  0 and Hz =0 (TM modes), and . Ez  0 and Hz 0 (HE modes).

**11.3.1.1 Field components for TM wave**

A transverse magnetic (TM) modes in a rectangular wave guide are characterized Hz = 0 and Ez  0. The Helmholtz wave equation of a TM wave for propagation of the electric field in a lossless medium can be written is given by



Expanding this equation for our z-propagating fields we have

 (11.26)

This is a partial differential equation which can be solved to get the different field components Ex, Hx, Ey, and Hy. . To solve this equation, we employ the method of separation of variable, by assuming Ez = XY

Here Ez can be expressed as the product of a function X, which only depends on x, and a function Y, which only depends on y.

Where, X is a pure function of ‘x’ only

Y is a pure function of ‘y’ only

Since X and Y are independent variables,

 (11.27)

 (11.28)

Using these two equations in equation (11.26), we get

 (11.29)

Dividing throughout by XY, we get

 (11.30)

 is a pure function of x only

 is a pure function of y only

The sum of these terms is constant. Hence each term must be equal to a constant separately since X and Y are independent variables. We use separation of variables method to solve the differential equation (11.30).

Let  (11.31)

and  (11.32)

Where -A2 and -B2 are constants

Substituting Equations (11.31) and (11.32) in equation (11.30). We get –B2 -A2 + h2  =0

h2= A2+B2  (11.33)

Equations (11.31) and (11.32) are ordinary 2nd order differential equations, the differential equation has the general solution, and the solutions of above equation are given by,

X = C1 cos Bx +C2 sin Bx (11.34)

Y = C3 cos Ay+C4 sin Ay (11.35)

Where C1, C2, C3, and C4, are constantswhich can be evaluated by applying boundary conditions

The complete solution is given by Ez = XY

Substituting values of equations of X and Y we get the total solution of Helmholtz equation in rectangular coordinates is

Ez = [C1 cos Bx +C2 sin Bx][C3 cos Ay+C4 sin Ay] (11.36)

**Boundary conditions**

Since the entire surface of the rectangular waveguide acts as a short circuit or ground for electric field, Ez=0 all along the boundary walls of the waveguide. Since there are four walls, as shown in figure 11.4 there are four boundary conditions.

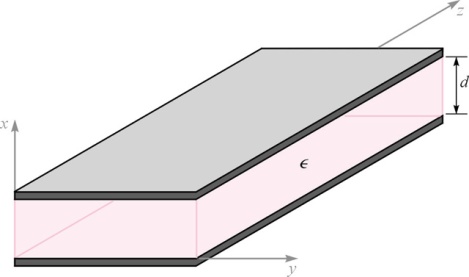


Figure 11.4

**1st boundary condition: -** [bottom plane or bottom wall]

We know that Ez =0 all along the bottom wall i.e Ez =0 at y=0x0 to a,

Stands for “for all” and x0 to a means x varying between 0 to a,

**2nd boundary condition: -** [left side plane or left side wall]

Ez =0 at x=0y0 to b

**3rd boundary condition: -** [Top plane or top wall]

Ez =0 at y=bx0 to a

**4th boundary condition: -** [Right side plane or right side wall]

Ez =0 at x=ay0 to b

1. Substituting 1st boundary conditions in equation (11.36), given by

We have Ez =0 at y=0x0 to a

0= [C1 cos Bx +C2 sin Bx][C3 cos 0+C4 sin 0]

0= [C1 cos Bx +C2 sin Bx]C3 ( since cos0 = 1, sin0=0)

This is true for all x0 to a

C1 cos Bx +C2 sin Bx 0 Therefore C3 = 0

Now substituting C3 = 0 in equation (11.36) the solution reduced to

Ez = [C1 cos Bx +C2 sin Bx][C4 sin Ay] (11.37)

1. Substituting 2st boundary conditions in equation (11.37), given by

We have Ez =0 at x=0y0 to b

0= [C1 cos 0 +C2 sin 0][C4 sin Ay]

0= C1 C4 sin Ay (since cos0 = 1, sin0 = 0)

This is true for all y0 to b

sin Ay  0 and C4  0thereforeC1 ­= 0

Now substituting C1 = 0 in equation (11.37) the solution further reduced to

Ez = C2 C4 sin Bxsin Ay (11.38)

1. Substituting 3rd boundary conditions in equation (11.38), given by

We have Ez =0 at y=bx0 to a

0= C2 C4 sin Bxsin Ab

This is true for all x0 to a, since sin Bx 0 C4 0 C20 therefore

sin Ab = 0

Ab = nπ where n is a constant, n = 0, 1, 2…

Therefore A =  (11.39)

1. Substituting 4th boundary conditions in equation (11.38), given by

We have Ez =0 at x=ay0 to b

0= C2 C4 sin Basin Ay

This is true for all y0 to b, since sin Ay 0 C4 0 C20 therefore

sin Ba = 0

Ba = mπ where n is a constant, m = 0, 1, 2…

Therefore B =  (11.40)

Now the complete solution is given by substituting A and B values in equation (11.38)

Ez = C2 C4 sin xsin y 

Where  = Propagation along the z-direction

 = sinusoidal variation w.r.t.‘t’

The general solution for the z-directed electric field for TM mode propagation is therefore.

Ez = Csin xsin y 

Where C is the product of the C2 and C4 constants

We can find the transverse field components by using equations (11.22) to (11.25) yields the TM field equations in rectangular waveguides as

 (11.41)

 (11.42)

 (11.43)

 (11.44)

**Modes of TM wave in Rectangular Waveguide**

Transverse Magnetic (TM) means the magnetic field is perpendicular to the propagation direction. A transverse magnetic (TM) modes have Hz = 0 and Ez  0. In TM mode, the magnetic lines of flux are perpendicular to the axis of the waveguide.The *m* and *n* represent the mode of propagation and indicates the number of variations of the field in the *x* and *y* directionsNote that for the TM mode, if *n* or *m* is zero, all fields are zero. The order of the next modes change depending on the dimensions of the guide

Depending on the values of m and n, we have various modes in TM waves. In general we represent the modes as TMmn­ where m and n are defined earlier.

**Various TMmn modes**

**TM00 mode:** m = 0 and n = 0

For m = 0 and n = 0, i.e. the number of half wave variations on wide dimension and narrow dimension are zero, therefore all the field components vanish inside the wave guide, therefore TM00 mode cannot exist.

**TM01 mode:** m = 0 and n = 1

For m = 0 and n = 1, i.e. there is only one half wave variation of magnetic field along the wide dimension and there is no magnetic field variation along the narrow dimension, therefore again all the field components vanish inside the wave guide, therefore TM01 mode cannot exist.

**TM10 mode:** m = 1 and n = 0

For m = 1 and n = 0, i.e. there is only one half wave variation of magnetic field along the narrow dimension and there is no magnetic field variation along the wide dimension, therefore again all the field components vanish inside the wave guide, therefore TM10 mode cannot exist.

**TM11 mode:** m = 1 and n = 1

For m = 1 and n = 1, i.e. there is only one half wave variation of magnetic field along the narrow dimension and wide dimension, now we have all the four components Ex, Ey, Hx, and Hy inside the wave guide, therefore TM11 mode exist and for all higher values of m and n, the components exist i.e. all higher modes do exist.

**11.3.1.2 Field components for TE wave**

A transverse electric (TE) modes in a rectangular wave guide are characterized Ez = 0 and Hz  0. The Helmholtz wave equation of a TE wave for propagation of the electric field in a lossless medium can be written is given by



Expanding this equation for our z-propagating fields we have

**** (11.45)

This is a partial differential equation which can be solved to get the different field components Ex, Hx, Ey, and Hy. . To solve this equation, we employ the method of separation of variable, by assuming Hz = XY

Here Hz can be expressed as the product of a function X, which only depends on x, and a function Y, which only depends on y.

Where, X is a pure function of ‘x’ only

Y is a pure function of ‘y’ only

Since X and Y are independent variables,

 (11.46)

 (11.47)

Using these two equations in equation (11.26), we get

 (11.48)

Dividing throughout by XY, we get

 (11.49)

 is a pure function of x only

 is a pure function of y only

The sum of these terms is constant. Hence each term must be equal to a constant separately since X and Y are independent variables. We use separation of variables method to solve the differential equation (11.49).

Let  (11.50)

and  (11.51)

Where -A2 and -B2 are constants

Substituting Equations (11.31) and (11.32) in equation (11.30). We get –B2 -A2 + h2  =0

h2 =A2+B2  (11.52)

Equations (11.50) and (11.51) are ordinary 2nd order differential equations, the differential equation has the general solution, and the solutions of above equation are given by

X = C1 cos Bx +C2 sin Bx (11.53)

Y = C3 cos Ay+C4 sin Ay (11.54)

Where C1, C2, C3, and C4, are constantswhich can be evaluated by applying boundary conditions

The complete solution is given by Hz = XY

Substituting values of equations of X and Y we get the total solution of Helmholtz equation in rectangular coordinates is

Hz = [C1 cos Bx +C2 sin Bx][C3 cos Ay+C4 sin Ay] (11.55)

If only looking at the wave traveling in z direction

**Boundary conditions**

Since the entire surface of the rectangular waveguide acts as a short circuit or ground for electric field, Hz=0 all along the boundary walls of the waveguide. Since there are four walls, as shown in figure 11.5 there are four boundary conditions.

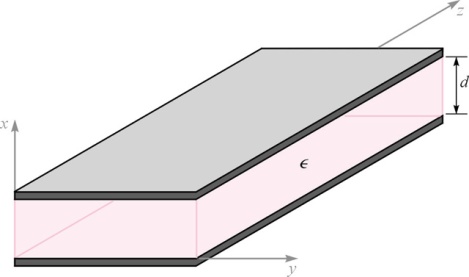


Figure 11.5

Here since we are considering a TE wave

Ez = 0 but we have components along x and y direction

Ex = 0 all along bottom and top walls of the waveguide

Ey = 0 all along left and right walls of the waveguide

**1st boundary condition: -**

Ex = 0 at y=0x0 to a (bottom wall)

**2nd boundary condition: -**

Ex =0 at y=bx0 to a (top wall)

**3rd boundary condition: -**

Ey =0 at x=0y0 to b (left side wall)

**4th boundary condition: -**

Ey =0 at x=ay0 to b (right side wall)

1. Substituting 1st boundary conditions in equation (11.55), given by

We have Ex =0 at y=0x0 to a Let us write Ex in terms of Hz

From equation (11.22), we have



Since Ez = 0, the 1st term in above equation is zero therefore





Now substituting 1st boundary condition in the above equation we get



Since (C1 cos Bx +C2 sin Bx) 0 Therefore C4 = 0.

Substituting the value of C4­ in equation (11.55) to reduce the solution,

Hz = (C1 cos Bx +C2 sin Bx)(C3 cos Ay) (11.56)

1. Substituting 3rd boundary conditions in equation (11.56), given by

We have Ey =0 at x=0y0 to b

From equation (11.23) we have



Since Ez = 0 and substituting the value of Hz from equation (11.56) we get





Now substituting 3rd boundary condition in the above equation we get



Since cos Ay 0, B0, and C30, therefore C2 = 0.

Substituting the value of C2­ in equation (11.56) to reduce the solution,

Hz = C1C3 cos Bxcos Ay (11.57)

1. Substituting 2nd  boundary conditions

Ex =0 at y=bx0 to a

From equation (11.22) we have



Since Ez = 0 and substituting the value of Hz from equation (11.57) we get

 (Since Ez=0)



Now substituting 3rd boundary condition in the above equation we get



Since cos Bx 0, C1 0, and C3 0 therefore

sin Ab = 0

Ab = nπ where n is a constant, n = 0, 1, 2…

Therefore A =  (11.58)

1. Substituting 4th boundary conditions

Ey =0 at x=ay0 to b

From equation (11.23) we have



Since Ez = 0 and substituting the value of Hz from equation (11.57) we get





Now substituting 4th boundary condition in the above equation we get



since cos Ay 0 C1 0 C30 therefore

sin Ba = 0

Ba = mπ where n is a constant, m = 0, 1, 2…

Therefore B =  (11.59)

Now the complete solution is given by By substituting A and B values in equation (11.57)

Hz = C1 C3 cos xsin y 

Where  = Propagation along the z-direction

 = sinusoidal variation w.r.t.‘t’

Let C = C1 C3, some other constant

The general solution for the z-directed electric field for TM mode propagation is therefore.

Hz = Ccos xsin y 

We can find the transverse field components by using equations (11.22) to (11.25) yields the TE field equations in rectangular waveguides as

 (11.60)

 (11.61)

 (11.62)

 (11.63)

**Modes of TE wave in Rectangular Waveguide**

In the TE modes, the electric field is transverse (or normal) to the direction of wave propagation.

A transverse electric (TE) wave has Ez = 0 and Hz = 0. In TE mode, the electric lines of flux are perpendicular to the axis of the waveguide. The *m* and *n* represent the mode of propagation and indicates the number of variations of the field in the *x* and *y* directions.Note that for the TE mode, if *n* or *m* is zero, all fields are zero. The order of the next modes change depending on the dimensions of the guide

Depending on the values of m and n, we have various modes in TE waves. In general we represent the modes as TEmn­ where m and n are defined earlier.

**Various TEmn modes**

**TE00 mode:** m = 0 and n = 0

For m = 0 and n = 0, i.e. the number of half wave variations on wide dimension and narrow dimension are zero, therefore all the field components vanish inside the wave guide, therefore TE00 mode cannot exist.

**TE01 mode:** m = 0 and n = 1

For m = 0 and n = 1, i.e. there is only one half wave variation of electric field along the wide dimension and there is no electric field variation along the narrow dimension, therefore the field components Ey, = 0, Hx, = 0, Ex and Hy exist, therefore TE01 mode can exist.

**TE10 mode:** m = 1 and n = 0

For m = 1 and n = 0, i.e. there is only one half wave variation of electric field along the narrow dimension and there is no electric field variation along the wide dimension, therefore the field components Ex, = 0, Hy, = 0, Ey and Hx exist, therefore TE10 mode can exist.

**TE11 mode:** m = 1 and n = 1

For m = 1 and n = 1, i.e. there is only one half wave variation of electric field along the narrow dimension and wide dimension, now we have all the four components Ex, Ey, Hx, and Hy inside the wave guide, therefore TM11 mode exist and for all higher values of m and n, the components exist i.e. all higher modes do exist.

**11.3.1.3 Impossibility of TEM waves**

The term TEM (**T**ransverse **E**lectro **M**agnetic) also known as **T**ransverse **E**lectric and **M**agnetic, refers to a condition in which both the electric and magnetic fields are parallel to a boundary plane and there are no longitudinal components of either field.

The transverse electric and magnetic (TEM) modes are characterized by Ez = 0 and Hz = 0. In other words, there is no cutoff frequency for waveguides that support TEM waves. This is the transverse electromagnetic (TEM) mode, in which both the E and H fields are transverse to the direction of wave propagation. ( Ez = 0 and Hz = 0). Substituting these values in equations (11.22) to (11.25) all the field components along x and y directions Ex, Ey, Hx, and Hy vanish and hence a TEM wave cannot exist inside wave guide

Suppose a TEM wave exists within a hollow guide of any shape, then line of magnetic field H lies entirely in the transverse plane. Also in a non-magnetic material

This means that lines of H should be closed loops. Therefore if a TEM wave exists inside the guide, the lines of H will be closed loops in a plane perpendicular to that axis. Now by Maxwell’s first equation, the magneto motive force around each of these closed loops must equal the axial current (conduction or displacement) through the loop. In the case of a guide with inner conductor e.g. a coaxial line, this axial current through H loops is the conduction current in the inner conductor. However, for a hollow wave-guide having no inner conductor, this axial current must be displacement current. But an axial- displacement current requires an axial component of E, something not present in a TEM wave. Hence the TEM wave cannot exist in a single conductor wave-guide.

Rectangular, circular, elliptical, and all hollow, metallic waveguides cannot support TEM waves. It can be shown that at least two separate conductors are required for TEM waves. Examples of waveguides that will allow TEM modes include coaxial cable, parallel plate waveguide, strip line, and micro strip. Micro strip is the type of microwave circuit interconnection that we will use in this course. This “waveguide” will support the “quasi-TEM” mode, which like regular TEM modes has no nonzero cutoff frequency. However, if the frequency is large enough, other modes will begin to propagate on a micro strip. This is usually not a desirable situation.

11.3.2 Cut-off Frequencies of rectangular waveguide

The cutoff frequency of an [electromagnetic waveguide](http://en.wikipedia.org/wiki/Waveguide_%28electromagnetism%29) is the lowest frequency for which a mode will propagate in it. The cutoff frequency is found with the [characteristic equation](http://en.wikipedia.org/wiki/Characteristic_equation) of the [Helmholtz equation](http://en.wikipedia.org/wiki/Helmholtz_equation) for electromagnetic waves, which is derived from the [electromagnetic wave equation](http://en.wikipedia.org/wiki/Electromagnetic_wave_equation) by setting the longitudinal [wave number](http://en.wikipedia.org/wiki/Wave_number) equal to zero and solving for the frequency.

The **cutoff frequency** is the **operating** frequency below which attenuation occurs and above which propagation lakes place.

The cutoff frequency is the frequency at which all lower frequencies are attenuated by the waveguide, and above the cutoff frequency all higher frequencies propagate within the waveguide. The cutoff frequency defines the high-pass filter characteristic of the waveguide, above this frequency, the waveguide passes power, and below this frequency the waveguide attenuates or blocks power. The cutoff frequency depends on the shape and size of the cross section of the waveguide.

From the relation h2 =  and also h2 =  A2+B2  therefore

h2 =  = A2+B2  (11.64)

Now substitute the values of A and B in above equation we get

h2 =  = A2+B2  =  (11.65)

= 

=  (11.66)

At lower frequencies 

i.e. 

γ then becomes real and positive and equal to the attenuation constant ‘α’ i.e. the wave is completely attenuated and there is no phase change. Hence the wave cannot propagate.

However at higher frequencies 

i.e. 

γ becomes imaginary, there will be phase change β and hence the wave propagates. At the transition, γ becomes zero and the propagation just starts. The frequency at which γ just becomes zero is defined as the cutoff frequency (or threshold frequency) ‘ fc’.

At f = fc , γ = 0 or ω = 2πf = 2πfc = ωc

Therefore 0 = 

ωc =  (11.67)

The expression for the cutoff frequencies & cutoff wavelengths in a rectangular waveguide

fc = 

fc =  (11.68)

fc =  since c =

fc =  (11.69)

The cutoff wavelength 𝝀c is

 =    (11.70)

Or  (11.71)

All wavelengths greater than 𝝀c are attenuated and those less than 𝝀c

are transmitted

**11.3.3 Filter Characteristics**

**11.3.4 Wave Impedance in rectangular wave guides**

We looked at a term called *characteristic impedance* when we were examining other forms of transmission lines. For waveguide there is a corresponding term called *characteristic wave impedance*. The wave impedance of a waveguide is defined as the ratio of the strength of electric field in one direction and the magnetic field along the other transverse direction at a certain point in the waveguide.



 (11.72)

Or  (11.73)

1. Wave impedance for a TM wave in rectangular waveguide:

 (11.74)

For a TM wave Hz = 0 and γ = jβ

or



We know that 





Where  is the intrinsic impedance of free space.

 (11.76)

Since λ0 is always less than λc for wave propagation .

this shows the wave impedance for a TM wave is always less than free space impedance.

1. Wave impedance of TE waves in rectangular waveguide.

 (11.77)

For a TE wave Ez = 0 and γ = jβ

 (11.78)

 (11.79)

Or  (11.80)

therefore ZTE > η as λ0 < λc for wave propagation. This shows that wave impedance for a TE wave is always greater than free space impedance.

For TEM waves between parallel planes or an ordinary parallel wire or co-axial transmission lines the cut-off frequency is zero and wave impedance for TEM wave is the free space impedance itself.

i.e., Z(TEM) = η

also when the waveguide has a dielectric other than air say with a dielectric constant εr then , the behavior of the waveguide gets changed.

For air dielectric, we know the guide wavelength is given by

 (11.81)

For the case of waveguide with dielectric constant εr,

 (11.82a)

and  (11.82b)

Since εr > 1 and λdielectric < λair, hence frequencies less than cut-off values can pass through the same guide.

**11.3.5 Dominant mode and Degenerate modes**

As already discussed, the walls of the waveguides can be considered as nearly perfect conductors. Therefore the boundary conditions require that electric field be normal i.e., perpendicular, to the waveguide walls. The magnetic fields must be tangential i.e., parallel, to the waveguide walls. Because of these boundary conditions a zero subscript can exist in the TE mode but not in the TM mode. For e.g. TE10, TE01, TE20 etc. modes can exist in a rectangular waveguide but only the TM11, TM12, TM21 etc. modes can exist. Also the cut-off frequency relationship shows that the physical size of the waveguide determines the propagation of modes depending on the values of m and n. the minimum cut-off frequency for a rectangular waveguide is obtained for a dimension a>b for m=1 and n=0, i.e., TE10 mode is the dominant mode for a rectangular waveguide. (since for TEmn modes m≠0 or n≠0, the lowest order mode TE10 is the dominant mode for a>b).

Some of the higher order modes, having the same cut-off frequency are called degenerate modes. For a rectangular waveguide TEmn/ TMmn modes for which both m≠0, n≠0 will always be degenerate modes. For a square guide in which a=b, all the TEpq, TEqp, TMpq and TMqp modes are together degenerate modes. It is necessary that higher order degenerate modes are not supported by the guide in the operating band of frequencies to avoid undesirable components appearing at the output along with losses.

**11.3.6 Sketches of TE and TM mode fields in the cross-section**

As mentioned before, the integers *m* and *n* indicate the number of half-cycle variations in the *x-y* cross section of the guide. Thus for a fixed time, the field configuration of Figure 11.56 results for TM21, mode, for example

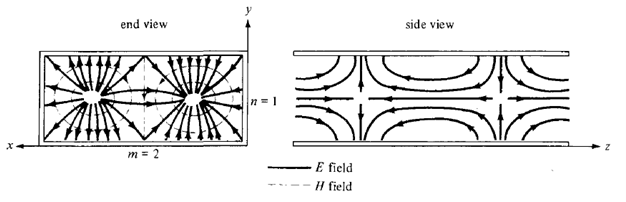


Figure 11.56Field Configuration for TE21 mode.

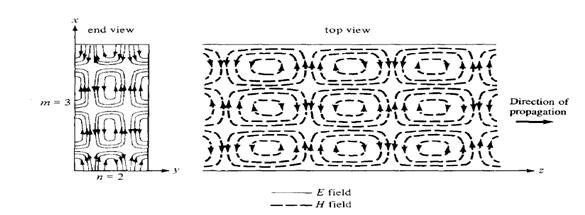


Figure 11.57 Field configurations for TE32 mode

**11.3.7 Mode Characteristics - Phase Velocity (Vp) and Group Velocity (Vg)**

The phase velocity **(Vp)** of a waveguide is defined as the rate at which the wave changes its phase in terms of the guide wavelength per unit time

The velocity of propagation of equi-phase surfaces along a guide

i.e., 

i.e.,  (11.83)

where , .

Expression for **Vp**

The relation h2 =  and also h2 =  A2+B2  therefore

h2 =  = A2+B2

Now substitute the values of A and B in above equation we get

h2 =  = A2+B2  =  (11.84)

= 

At f = fc , γ = 0 or ω = 2πf = 2πfc = ωc

Therefore 0 = 





This can be written as

 **** 





 (11.85)

i.e.,  (11.86)

we also know that, f(any frequency)=c/λ0, where λ0 is free space wavelength and fc (cut-off frequency)=c/λc, where λc is cut-off wavelength



** (11.87)**

** (11.88)**

**Group Velocity (Vg)**

If there is modulation in the carrier, the modulation envelope actually travels at velocity slower than that of carrier alone and of course slower than speed of light. The velocity of modulation envelope is called the group velocity (Vg). This happens when a modulated signal travels in a waveguide, the modulation goes on slipping backward with respect to the carrier.

It is defined as the rate at which the wave propagates through the waveguide and is given by

 (11.89)

We know 

Now differentiating β w.r.t. ‘ω’, we get





Or

 (11.90)

Or

 (11.91)

Consider the product of Vp and Vg

i.e., 

 (11.92)

**11.3.8 Power transmission in a rectangular waveguide**

The power transmitted through a waveguide and the power loss in the guide walls can be calculated by means of complex pointing theorem. We assume that the guide is terminated in such a way that there is no reflection from the receiving end or that the waveguide is infinitely long as compared with its wavelength.

The power transmitted Ptr, through a waveguide given by,

 (11.93)

For a lossless dielectric, the time average power flow through a rectangular waveguide is

 (11.94)

Where 

 (11.95)

 (11.96)

for TMmn mode, the average power transmitted through a rectangular waveguide of dimensions *a* and *b* is

 (11.97)

for TEmn mode, 

** (11.98)**

**11.3.9 Power losses in a rectangular waveguide**

Losses in a waveguide can be due to attenuation below cut-off and losses associated with attenuation due to dissipation within the waveguide walls and the dielectric within the waveguide.

At frequencies below the cut-off frequency (f<fc), the propagation constant ‘γ’ will have only the attenuation term ‘α’. (γ=α+jβ) that is to say that the phase constant β itself becomes imaginary implying wave attenuation.

Hence



We know that  (11.99)

Therefore, 

Hence the cut-off attenuation constant α is given by

 dB/length. (11.100)

In fact this is the stop band attenuation of the waveguide high pass filter. For f > fc, the waveguide exhibits very low loss and for f < fc, the attenuation is high and results in full reflection of the wave i.e., cut-off attenuation is basically the reflection loss.

Attenuation constant due to an imperfect, nonmagnetic dielectric in the waveguide is given by

 dB/length (11.101)

Where tan δ = dielectric loss tangent of the insulating material (dielectric).

The attenuation constant due to the imperfect conducting walls for TE10 mode is given by

 Np/length (11.102)

Where Rs is the sheet resistivity in ohm/m2

η0 = intrinsic impedance of free space (377 Ω)

thus  (11.103)

where σ is the conductivity of the metallic walls in S/m and the skin depth is

 (11.104)

Where f = frequency

μr = relative permeability (typically it is 1)

μ0 = permeability of free space (4π X 10-7 H/m)

for example, waveguide is constructed from aluminum which has a conductivity of 3.54X107 S/m.

therefore dB/m (11.105)

**Solved Examples:**

**Example 11.1:** A rectangular waveguide has cross sectional area of 2.29 cm X 1.45 cm and the operating frequency is 9 GHz.

Calculate:

1. Free space wavelength (ii) Cut-off wavelength
2. Cut-off frequency (iv) Angle of incidence

(v) Guide wavelength (vi) Phase velocity

1. Phase shift constant (viii) Wave impedance of the guide.

**Solution:**

1. Free space wavelength, 
2. Cut-off wavelength, 
3. Cut-off frequency, 
4. Angle of incidence, 
5. Guide wavelength, 
6. Phase velocity in the guide, 
7. Phase shift constant, 
8. Wave impedance of the waveguide,

for TE mode, 

For TM mode 

**-----------------------------------------------------**

**Example** 11.2: Calculate the cutoff frequency for the first 8 modes of a waveguide that has a = 0.900 inches and b = 0.600 inches.

Solution: a = 0.900 in = 0.02286 m, b= 0.600 in = 0.01524 m



For air-filled guide we have

Evaluating all combinations of modes for m = 0,1,2,3 and n = 0,1,2,3 we find

**Mode fcmn(GHz)**

TE10  6.56

TE01 9.84

TE11 11.83

TM11 11.83

TE20 13.12

TE21 16.40

TM30 19.69

TE02 19.69

**Example 11.3** An air filled rectangular waveguide has dimension a=6cm and b=4cm. the signal frequency is 4GHz. Compute the following for the TE10, TE01, TE11 and TM11modes.

1. Cut-off frequency
2. Wavelength in the waveguide
3. Phase constant and phase velocity in the waveguide and
4. Group velocity and wave impedance on the waveguide.

**Solution**: (a) cut-off frequency for TEmn and TMmn modes is



For TE10 mode the cut-off frequency is



For TE01 mode the cut-off frequency is



For TE11 and TM11 mode the cut-off frequency is,



As the signal frequency of 4 GHz is below fc for TE11 and TM11, it will be attenuated.

(b) Wavelength in the waveguide for both TE and TM modes

λg for TE10 

TE01

This means that the wave decays exponentially and is attenuated as fc > f. the same will be the case for TE11 and TM11.

(c) Phase constant β for



There will be phase constant only for propagating waves and hence frictions value for TE01, TE11 and TM11.

**Phase Velocity**

****

Phase velocity is higher than velocity of light.

(d) group velocity vg for



Group velocity is less than the velocity of light.

There is no group velocity for TE01, TE11, TE12 as the wave is attenuated.

Wave impedance



**Example 11.4**: Calculate the cut-off frequency of the following modes in a square waveguide 4 cm X 4 cm TE10, TM11 and TE22.

**Solution:** a square waveguide is a special case of rectangular waveguide, where a = b.

For TE10, λc = 2a = 8 cm;



For TE11, 



For TE22, 

**Example 11.5:** when the dominant mode is propagated in an air-filled rectangular waveguide, the guide wavelength for a frequency of 10,000 MHz is 4 cm. Calculate the breadth of the guide?

**Solution:** For a rectangular waveguide, the dominant mode is the TE10 mode. TE10 mode can propagate at a lower frequency.

given f = 10,000 MHz = 10 GHz; λg=4cm

we know for TE10 mode λg = 2a



Substituting the value of λg and λ0 in the above equation, we get

 or 

Squaring both sides, we get









λc > λ0 the wave propagates and

λc = 2a for TE10 mode





**11.4 Circular Waveguide**

A circular waveguide ia basically a tubular, circular conductor. An example is a rotating joint which transmits an electromagnetic wave to the feeder of a rotating radar antenna. The problem of these waveguides is that due to their geometry they do not keep the plane of the polarization of the transmitted mode when the waveguide is long. The frequency band of the single mode operation of a circular waveguide is narrower than the same band of a rectangular waveguide. We have to use the cylindrical coordinate system. The field in a circular waveguide can be separated into TE and TM modes, which are treated separately.

Figure 11.8 shows a circular waveguide of inner radius *ρ=a* and length *l*.

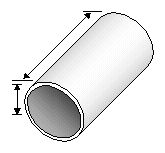


Fig. 11.8 Circular Waveguide

Here *φ* varies from *0* to *2π*, *ρ* varies from *0* to *a* and *l* varies along z-axis.

The Helmholtz’s wave equation for a TE and TM wave travelling in z direction in a circular waveguide are given by

 (11.106)

**11.4.1 Field components for TE wave**

For TE wave to propagate 

From Maxwell’s equation

 (11.107)

Expanding  in cylindrical coordinates

 (11.108)

We know  ( an operator)

 (11.109)

But 

Then the above equation becomes

 (11.110)

This is a partial differential equation, whose solution by separation of variables method is assumed to be

** (11.111)**

Where, P is a function of ρ only and

Q is a function of φ only.

Substituting for Hz in the equation (11.110), it reduces to

 (11.112)

**Or**

 (11.113)

Multiplying throughout the equation by , we get

 (11.114)

1. (2) (3) (4)

the terms (1), (2) and (4) of the above equation are function of ρ only and the term (3) is a function of φ only.

Let , where n2 is a constant. The equation (11.114) becomes

 (11.115)

Multiplying the throughout by P, we get

 (11.116)

This is similar to the Bessel’s equation of the form

 (11.117)

Whose solution is where  represents the nth order Bessel function of the first kind and Cn is a constant. To bring equation the above equation into standard Bessel form, we write,

 (11.118)

Therefore the solution of this equation is given by

 (11.119)

Also,  (11.120)

The general solution of this equation is given by

 (11.121)

Therefore the complete solution becomes as per ****  or

 (11.122)

where and 

** (where  )** (11.123)

If we consider a sinusoidal variation along ‘z’

**** (11.124)

**Boundary Conditions**

Now applying boundary conditions, we know that all along the surface of the circular waveguide at for all values of varying between 0 to 2π,

i.e.,  (11.125)

this implies 

here the prime denotes differentiation with respect to *ah*. The mth root of this equation is denoted by which are the eigen values given by

 (11.126)

Various root values are listed in Table 11.2, the equation (11.124) reduces to

**** (11.127)

Where and this equation represents all possible solutions of Hz for TEmn waves in circular waveguide. Since Jn are oscillatory functions, the are also oscillatory functions.

**Table 11.2 values for TEmn modes in circular waveguide**

|  |  |  |  |
| --- | --- | --- | --- |
| m  n | 1 | 2 | 3 |
| 0  1  2  3 | 3.832  1.841  3.054  4.201 | 7.016  5.331  6.706  8.015 | 10.173  8.536  9.969  11.346 |

The permissible values of *h* are given from eq. (11.126) as



The various field components Eρ, Eφ, Ez, Hρ, Hφ and Hz can be obtained by using the cylindrical coordinates, (as we have seen in case of rectangular waveguides) by using Maxwell’s curl equations. The field components are given by

 (11.128)

 (11.129)

 (11.130)

 (11.131)

 (11.132)

 (11.133)

Where 

Substituting for Hz in above equations with , the complete field equations for TEnm modes in circular waveguides can be shown to be

 (11.134)

 (11.135)

 (11.136)

 (11.137)

 (11.138)

 (11.139)

Where , the wave impedance in the guide where n=0,1,2,3,….. and m=1,2,3,4,…

**11.4.2 Field components for TM wave**

For a TM wave to propagate in a circular waveguide Hz=0 and Ez≠0. Hence the Helmholtz wave equation is given by



The solution of this equation (similar to TE waves ) is given by



By applying boundary conditions i.e., Ez=0 at ρ=a, we get,



Since Jn(ah) are oscillatory functions, there are infinite number of roots for which are called eigen values and are denoted by Pnm where Pnm=ah. A table for these roots for some values of *n* and *m* are shown in table 11.3.

**Table 11.3values for TMmn modes in circular waveguide**

|  |  |  |  |
| --- | --- | --- | --- |
| m  n | 1 | 2 | 3 |
| 0  1  2  3 | 2.405  3.832  5.135  6.380 | 5.520  7.106  8.417  9.761 | 8.645  10.173  11.620  13.015 |

The various field components are again obtained by using Maxwell’s curl equations by substituting.



The various field components are given by

 (11.140)

 (11.141)

 (11.142)

 (11.143)

 (11.144)

 (11.145)

**11.4.3 Characteristic equation and Cut-off frequency of a Circular Waveguide**

The cut-off wavelength is that mode for which the mode propagation constant, γ vanishes.

i.e., 

where  

where for TE wave and for TM waves.

Therefore for **TE wave**, the cut-off wavelength is given by

**** (11.146)

λc will be maximum if  is minimum.

And the cut-off frequency for TE wave can be written as

 =    (11.147)

Where ‘c’ is velocity of light.

Similarly for a **TM wave**



Where 

 (11.148)

And the cut-off frequency for TM wave can be written as

 (11.149)

**11.4.4 Dominant mode and Degenerate modes**

From the discussion above (Table 11.2 and 11.3), it is very clear that the lowest order cut-off frequency is obtained when =1.841 for n=1 and m=1 corresponding to TE11 mode. Hence dominant mode in circular waveguide is TE11.

If the fields exist (in both EZ≠0,HZ≠0),they would neither be transverse electric nor transverse magnetic, then we call such fields as Hybrid mode fields.

Also from these tables, we see that and hence all the TM0m and TM1m modes are degenerate in a uniform circular waveguide.

**Example 11.4.1:** for the dominant mode of operation in an air filled circular waveguide of inner diameter 4 cms, find (a) cut-off wavelength (b) cut-off frequency (c) wavelength in the guide?

Sol: the dominant mode in the circular waveguide is TE11, λc is maximum



It is given D=4 cms.



Since cut-off frequency is 4.395 GHz, frequencies higher than fc will be propagated. Assume a signal of frequency of 5 GHz is being propagated.



**Example 11.4.2** an air filled circular waveguide is to be operated at a frequency of 6 GHz and is to have dimensions such that fc=0.8f for TE11 mode. Determine (a) the diameter of the waveguide and (b) guide wavelength.

**Sol:** For TE11 mode in circular waveguide. Let r and D be the radius and the diameter of the waveguide respectively

(a)



It is given that fc=0.8f; and f= 6 GHz.

fc = 0.8 X 6 X 109=4.8GHz



(b) The guide wavelength is given by, 

 and λc = 6.25 cms



**Example 11.4.3** a TE11 wave is propagating through a circular waveguide. The diameter of the guide is 10 cm and the guide is air-filled. Find (a) cut-off frequency, (b) wavelength λg in the guide for frequency of 3 GHz, and (c) the wave impedance in the guide.

Sol: (a) the cut-off frequency of a circular waveguide is given by



Where ‘a’ is the radius of the guide; the diameter of the guide is given as 10 cm; therefore the radius r = 5 cm=0.05m

The value of Pnm for a TE11 wave guide is 1.841 (from table 11.2).



(b) Wavelength



(c) wave impedance



**11.5 Cavity resonators**

At low frequencies resonant circuits consists of Inductor (L) and Capacitor (C) in parallel or series. Frequency can be increased by reducing the values of L and C, since the frequency is inversely proportional to . Beyond one point L and C can’t be reduced further corresponds to highest frequency conventional circuit can oscillate. Cavity resonators are used in microwave circuits for high frequencies similarly as standard resonant LC circuits at lower frequencies.

Cavity resonator is a waveguide shorted at both ends.

1. When one end of waveguide is terminated in shorting plane, there will be reflections, hence standing waves.
2. When the other end of waveguide is also terminated with plane at distance, Signal bounces back and forth along Z axis between two shorting plates, results in resonance.
3. Hallow space is called Cavity Resonator.( i.e., Cavity Resonator is a waveguide shorted at both ends.
4. Modes of operations in a cavity are designated in terms of fields existing in X, Y and Z directions; m, n and p three subscripts are used.
5. General mode of propagation in cavity resonator is TEmnp or TMmnp.

Resonator generates a wave (or select specific frequency of a signal), while oscillator is an electronic circuit that creates a repetitive signal. An oscillatory system operates at microwave frequencies; it is the analogy of oscillator circuit.

**11.5.1 Types of Cavity resonators**

In microwave applications the commonly used cavity resonators are

* Circular Cavity Resonator
* Rectangular Cavity Resonator

**11.5.2 Resonant Frequency and dominant mode**

Let us first determine the resonant frequency of a general cavity resonator. We use the power balance for the reactive power. Assuming a shielded lossless structure without any radiation, the power balance for the reactive power is



Disconnecting the source we have **J** = 0 and we get the self-oscillation of the field in the volume. The condition for self-oscillation is determined by the equality of energies *We* = *Wm*, which is expressed by



The electric field can be determined from Maxwell’s first equation assuming σ= 0 and no external current. Then we have



Finally we get the resonant frequency of our cavity

 (11.150)

It follows from (9.1) that the resonant frequency of the cavity depends on the material parameters ***μ*** and ***ε*** and on the cavity geometry that is hidden in the integrals. This is common with standard LC resonant circuits. However,(9.1) shows that **the resonant frequency depends on the distribution of the electric and magnetic fields**. Consequently as we can excite an infinite number of particular modes in the cavity, we have an infinite number of resonant frequencies corresponding to these modes. Some modes are known as degenerated. These modes have a different field distribution but an equal resonant frequency.

**11.5.3 Circular and Rectangular Cavity Resonators**

**Circular Cavity Resonator**

Let us first determine the resonant frequency of the cavity resonator formed by a segment of any Waveguide terminated at both ends by ideally conducting planes. The planes are perpendicular to the longitudinal axis of the waveguide. The length of the segment is *d*, Fig. 11.9, so the conducting planes are located at *z* = 0 and *z* = *d*.

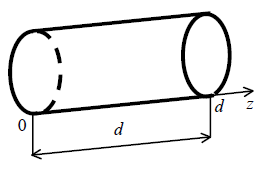


Fig.11. 9 Circular Cavity Resonator

It is reasonable to assume that the field distribution of the mode excited in this cavity originates from the distribution of the modes existing in the original waveguide forming the cavity. These modes have transversal and longitudinal propagation constants *kpmn* and *kzmn* which depend on two transversal modal numbers *m* and *n*. The distribution of the transversal components of its electric field is ***E****T0(x, y)* as the function of the two transversal coordinates *x*, *y*. The electric field in our cavity is now the superposition of the two waves traveling in the positive *z* direction and in the negative *z* direction

 (11.152)

This field must fulfill the two boundary conditions at *z* = 0 and *z* = *d*. At *z* = 0 we have

The field distribution is now

 (11.153)

At *z* = *d* we have



, p = 1,2,3,…. (11.154)

(9.3) tells us that we have the set of discrete values of the longitudinal propagation constants of the modes oscillating in the resonator. These constants depend on modal numbers *m*, *n* and *p*. Now we have the set of resonant frequencies

 (11.155)

In the case of a **cylindrical resonator** with radius *ro* we have to distinguish between the TE and TM modes, as their transversal propagation constants are different. For **TE modes**, *kpmn* is determined as  and the resonant frequency is

 (11.156)

For **TM modes**, *kpmn* is determined as and the resonant frequency is

 (11.157)

The dominant mode of the cylindrical waveguide is the TE11 mode, so the resonators are mostly designed to work with this mode, where α11 = 1.841

**Rectangular Cavity Resonator**

The geometry of a rectangular cavity is shown in Figure 11.10. It consists of a length d of rectangular waveguide shorted at both ends (z = 0, d).The particular value of the resonant frequency depends on transversal propagation constant *kpmn*, which is determined by the type of waveguide from which the resonator is composed. In the case of a **waveguide with a rectangular cross section** with dimensions *a, b* and the resonant frequency is

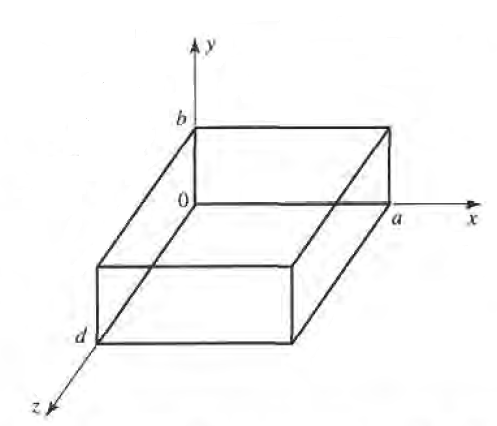


Fig. 11.10 A rectangular cavity resonator

 (11.158)

Where *kpmn is*



This formula is valid for both the TE and TM modes.

**11.5.4 Quality Factor (Q) and Coupling coefficient**

The quality factor Q is a measure of the frequency selectivity of a resonant or antiresonant circuit, and it is defined as

 (11.159)

Where W is the maximum stored energy and P is the average power loss.

At resonant frequency, the electric and magnetic energies are equal and in time quadrature. When the electric energy is maximum, the, the magnetic energy is zero and vice versa. The total energy stored in the resonator is obtained by integrating the energy density over the volume of the resonator:

 (11.160)

Where and are the peak values of the field intensities.

The average power loss in the resonator can be evaluated by integrating the power density over the inner surface of the resonator. Hence

 (11.161)

Here Ht is the peak value of the tangential magnetic intensity and Rs is the surface resistance of the resonator.

Substitution of equations (11.160) and (11.161) in (11.159) yields

 (11.162)

Since the peak value of the magnetic intensity is related to its tangential and normal components by 



Where Hn is the peak value of the normal magnetic intensity, the value of at the resonator walls is approximately twice the value of  averaged over the volume. So the Q of a cavity resonator as shown in Eq. (11.162) can be expressed approximately by

 (11.163)

An unloaded resonator can be represented by either a series or a parallel resonant circuit. The resonant frequency and the unloaded Q0 of a cavity resonator are

 (11.164)

 (11.165)

**Coupling Coefficient**

If the cavity is coupled by means of an ideal N:1 transformer and a series inductance Ls to a generator having internal impedance Zg, then the coupling circuit and its equivalent are as shown in fig. 11.11(a) and (b).



Fig. 11.11 Cavity coupled to a generator. (a) Coupling circuit, (b) Equivalent circuit

The loaded Q*l* of the system is given by

 for  (11.166)

The coupling coefficient of the system is defined as

 (11.167)

And the loaded Q*l* would become

 (11.168)

Rearranging of Eq. (4-3-24) yields

 (11.169)

Where  is the external Q.

There are three types of coupling coefficients:

1. **Critical coupling**: if the resonator is matched to the generator, then K=1. The loaded Q*l*is given by

 (11.170)

1. **Over coupling**: if K >1, the cavity terminals are at a voltage maximum in the input line at resonance. The normalized impedance at the voltage maximum is the standing-wave ratio ρ. That is

K=ρ

The loaded Q*l* is given by

 (11.171)

1. **Under coupling**: if K<1, the cavity terminals are at a voltage minimum and the input terminal impedance is equal to the reciprocal of the standing-wave ratio. That is,

 (11.172)

The loaded Qt is given by

 (11.173)

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Example 11.1:** an air filled rectangular cavity resonator has a=d=2 cm, b=1 cm and operated in TE101 mode.

Calculte: (i) resonant frequency (ii) if the cavity is filled with dielectric of relative permittivity 2.5, what is the resonant frequency.

Solution:

1. 



1. For dielectric filled cavity, 

**-----------------------------------------------**

11.2 Design a resonator tuned to the resonant frequency 15 GHz for the mode TE101. The resonator is formed by the segment of the rectangular waveguide with the dimensions *a* =

30 mm, *b* = 15 mm. The resonator is filled with air.

The dominant mode of the rectangular waveguide has the simplest field distribution, so we design the resonator for this mode. Thus for mode TE101 we have the resonant frequency (11.158)

 ,

where *c* is the speed of light in a vacuum. From the above formula we can determine the necessary length *d*

**

The resonator must be 10.6 mm long

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Example 11.3:** Design a cylindrical resonator tuned to the resonant frequency 14 GHz for the mode TE111. The resonator is filled with air and its radius is *r*o = 8.2 mm. Applying the same procedure as in example 11.1 we get the resonator length

**

The resonator must be 16.67 mm long.

**Example 11.4**: A rectangular cavity resonator 10 X 8 X 6 cm. Find the resonating frequency for TE111 mode.

Sol: a=10 cm =0.10 m

b=8 cm = 0.08 m

d=6 cm = 0.06 m and

m=1

n=1

p=1

the resonant frequency for a rectangular cavity is given by,



For  the equation becomes





**Example Problem 11.5**

Find the first five resonances of an air-filled rectangular cavity with dimensions of *a* = 5 cm, *b* = 4 cm and *c* = 10 cm (*c* > *a* > *b)*.

**Solution**

The expression for frequency of rectangular cavity resonator is given by



The given dimensions of rectangular cavity resonator are a=5cm, b= 4cm and c= 10cm.

TEmnp modes m = 0,1,2,….. n = 0,1,2,…. P = 1,2,3,…..

TMmnp modes m = 1,2,3,….. n = 1,2,3,…. P = 0,1,2,…..

The first five resonances of rectangular cavity resonator are

 TE101

 TE011

 TE102

 TE110

 TE111 , TM11

-----------------------------------------------

**11.6 Example Problem**

For a cavity of dimensions; 3cm x 2cm x 7cm filled with air and made of copper (sc=5.8 x 107)

Find the resonant frequency?

**Solution**



**11.6 Micro strip Lines**

In recent years, with the introduction of Monolithic Microwave Integrated Circuits (MMICs), micro strip lines and coplanar strip lines have been used extensively, because they provide one free and accessible surface on which solid state devices can be placed. Up to now we discussed the conventional transmission lines in detail. All electrical and electronic devices with high-power output commonly use conventional lines, such as coaxial lines or waveguides, for power transmission. However, the microwave solid state device is usually fabricated as a semiconducting chip with a volume on the order of 0.008-0.08 mm3. The method of applying signals to the chips and extracting output power from them is entirely different from that used for vacuum-tube devices. Microwave integrated circuits with microstrip lines are commonly used with the chips. The microstrip line is also called an open-strip line.

Modes on microstrip line are only quasi-transverse electric and magnetic (TEM). Thus the theory of TEM-coupled lines applies only approximately. Radiation loss in microstrip lines is a problem; particularly at such discontinuities as short-circuit posts, corners, and so on. However, the use of thin, high-dielectric materials considerably reduces the radiation loss of the open strip. A microstrip line has an advantage over the balanced-strip line because the open strip has better interconnection features and easier fabrication.

**11.6.1 Characteristic impedance (Z0) of Microstrip Lines**

Microstrip lines are used extensively to interconnect high-speed logic circuits in digital computers because they can be fabricated by automated techniques and they provide the required uniform signal paths. Figure 11.12 shows cross sections of a microstrip line and a wire-over-ground line for purposes of comparison. In Fig. 11.12(a) you can see that the characteristic impedance of a microstrip line is a function of the strip-line width, the strip-line thickness, the distance between the line and the ground plane, and the homogeneous dielectric constant of the board material.

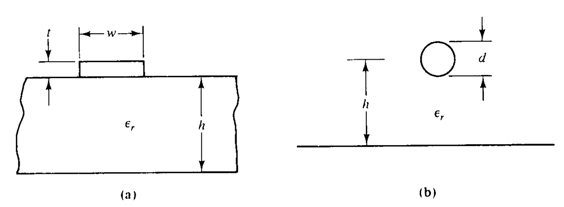


Figure 11.12 cross section of (a) a microstrip line and (b) a wire over ground line

Several different methods for determining the characteristic impedance of a microstrip line have been developed. The field-equation method was employed by several authors for calculating an accurate value of the characteristic impedance. However, it requires the use of a large digital computer and is extremely complicated. Another method is to derive the characteristic-impedance equation of a microstrip line from a well-known equation and make some changes. This method is called a comparative, or an indirect, method. The well-known equation of the characteristic impedance of a wire-over-ground transmission line, as shown in Fig. 11.12(b), is given by

  (11.174)

Where εr = dielectric constant of the ambient medium

h = the height from the center of the wire to the ground plane

d = diameter of the wire

If the effective or equivalent values of the relative dielectric constant *εr* of the ambient medium and the diameter *d* of the wire can be determined for the microstrip line, the characteristic impedance of the microstrip line can be calculated.

**11.6.2 Effective dielectric constant** (*εre* ): For a homogeneous dielectric medium, the propagation-delay time per unit length is

  (11.175)

where μ, is the permeability of the medium and ε is the permittivity of the medium. In free space, the propagation-delay time is

 (11.176)

where





In transmission lines used for interconnections, the relative permeability is 1. Consequently, the propagation-delay time for a line in a nonmagnetic medium is

 (11.177)

The effective relative dielectric constant for a microstrip line can be related to the relative dielectric constant of the board material. DiGiacomo and his coworkers discovered an empirical equation for the effective relative dielectric constant of a microstrip line by measuring the propagation-delay time and the relative dielectric constant of several board materials, such as fiberglass-epoxy and nylon phenolic.

The empirical equation, as shown in Fig. 11.12, is expressed as

 (11.178)

where εr  is the relative dielectric constant of the board material and εre is the effective relative dielectric constant for a microstrip line.

**Transformation of a rectangular conductor into an equivalent circular conductor**. The cross-section of a microstrip line is rectangular, so the rectangular conductor must be transformed into an equivalent circular conductor. Springfield discovered an empirical equation for the transformation. His equation is

 (11.179)

where d = diameter of the wire over ground

w = width of the microstrip line

t = thickness of the microstrip line

The limitation of the ratio of thickness to width is between 0.1 and 0.8, as indicated in Fig. (11.176).

**Characteristic impedance equation**. Substituting Eq. (11.178) for the dielectric constant and Eq. (11.179) for the equivalent diameter in Eq. (11.174) yields

 (11.180)

where εr = relative dielectric constant of the board material

h = height from the microstrip line to the ground

w = width of the microstrip line

t = thickness of the microstrip line

Equation (11.180)is the equation of characteristic impedance for a narrow microstrip line. The velocity of propagation is

 (11.181)

The characteristic impedance for a wide microstrip line was derived by Assadourian and others and is expressed by

 (11.182)

**11.6.3 Losses in Microstrip Lines**

Microstrip transmission lines consisting of a conductive ribbon attached to a dielectric sheet with conductive backing (see Fig.11.13) are widely used in both microwave and computer technology. Because such lines are easily fabricated by printed-circuit manufacturing techniques, they have economic and technical merit.



Fig. 11.13 schematic diagram of a microstrip line

The characteristic impedance and wave-propagation velocity of a microstrip line was analyzed in Section 11.6.1. The other characteristic of the microstrip line is its attenuation. The attenuation constant of the dominant microstrip mode depends on geometric factors, electrical properties of the substrate and conductors, and on the frequency.

For a nonmagnetic dielectric substrate, two types of losses occur in the dominant microstrip mode: (1) dielectric loss in the substrate and (2) ohmic skin loss in the strip conductor and the ground plane. The sum of these two losses may be expressed as losses per unit length in terms of an attenuation factor α. From ordinary transmission-line theory, the power carried by a wave traveling in the positive z direction is given by

 (11.183)

Where  is the power at z=0.

The attenuation constant α can be expressed as

 (11.184)

Where αd is the dielectric attenuation constant and αc is the ohmic attenuation constant.

Substitution of equation (11.183) in to equation (11.184) results in

 Np/cm (11.185)

And  Np/cm (11.186)

**Dielectric losses**. As stated in the earlier chapter, when the conductivity of a dielectric cannot be neglected, the electric and magnetic fields in the dielectric are no longer in time phase. In that case the dielectric attenuation constant, is given by

 Np/cm (11.187)

Where σ is the conductivity of the dielectric substrate board in .

**Ohmic losses.** In a microstrip line over a low-loss dielectric substrate, the predominant sources of losses at microwave frequencies are the nonperfect conductors. The current density in the conductors of a microstrip line is concentrated in a sheet that is approximately a skin depth thick inside the conductor surface and ex-posed to the electric field. Both the strip conductor thickness and the ground plane thickness are assumed to be at least three or four skin depths thick. The current density in the strip conductor and the ground conductor is not uniform in the transverse plane. The microstrip conductor contributes the major part of the ohmic loss. A diagram of the current density J for a microstrip line is shown in Fig. 11.13.

Because of mathematical complexity, exact expressions for the current density of a microstrip line with nonzero thickness have never been derived. Several researchers have assumed, for simplicity, that the current distribution is uniform and equal to I/w in both conductors and confined to the region |x| < w/2. With this assumption, the conducting attenuation constant of a wide microstrip line is given by

 dB/cm for  (11.188)

Where  is the surface skin resistance in Ω/square.

 is Ω/square.

 is the skin depth in cm.

**Radiation Losses:** In addition to the conductor and dielectric losses, microstrip line also has radiation losses. The radiation loss depends on the substrate's thickness and dielectric constant, as well as its geometry. Lewin has calculated the radiation loss for several discontinuities using the following approximations:

I. TEM transmission

2. Uniform dielectric in the neighborhood of the strip, equal in magnitude to an effective value

3. Neglect of radiation from the transverse electric (TE) field component parallel to the strip

4. Substrate thickness much less than the free-space wavelength

Lewin’s results show that the ratio of radiated power to total dissipated power for an open-circuited microstrip line is

 (11.189)

Where  is a radiation factor given by

 (11.190)

In which εre is the effective dielectric constant and λ0 = c/f is the free-space wave-length.

The radiation factor decreases with increasing substrate dielectric constant. So, alternatively, eq. (11.189) can be expressed as

 (11.191)

Where Rr is the radiation resistance of an open-circuited microstrip and is given by

 (11.192)

The ratio of the radiation resistance R, to the real part of the characteristic impedance Zo of the microstrip line is equal to a small fraction of the power radiated from a single open-circuit discontinuity. In view of Eq. (11.189), the radiation loss decreases when the characteristic impedance increases. For lower dielectric-constant substrates, radiation is significant at higher impedance levels. For higher dielectric-constant substrates, radiation becomes significant until very low impedance levels are reached.

**11.6.4 Quality Factor Q of Microstrip Lines**

Many microwave integrated circuits require very high quality resonant circuits. The quality factor Q of a microstrip line is very high, but it is limited by the radiation losses of the substrates and with low dielectric constant. Recall that for uniform cur-rent distribution in the microstrip line, the ohmic attenuation constant of a wide microstrip line is given by Eq. (11.188) as

 dB/cm

And that the characteristic impedance of a wide microstrip line given as

 Ω

The wavelength in the microstrip line is

 cm (11.193)

Where *f* is the frequency in GHz.

Since Qc is related to the conductor attenuation constant by

 (11.194)

Where αc is in dB/λg, Qc of a wide microstrip line expressed as

 (11.195)

Where h is measured in cm and Rs is expressed as

 Ω/square. (11.196)

Finally the quality factor Qc of a wide microstrip line is

 (11.197)

Where σ is the conductivity of the dielectric substrate board in .

Similarly, a quality factor Qd is related to the dielectric attenuation constant:

 (11.198)

Where αd is the attenuation constant per wavelength, in dB/λg, and is given as

 (11.199)

Where and *λ0* is the wavelength in free space, or

and *c* is the velocity of light in vacuum.

Substituting Eq. (11.199) into Eq. (11.198) yields

 (11.200)

where λ0 is the free-space wavelength in cm. Note that the Qd for the dielectric attenuation constant of a microstrip line is approximately the reciprocal of the dielectric loss tangent θ and is relatively constant with frequency.

**Example: Characteristic Impedance of a Microstrip Line**

A certain microstrip line has the following parameters: εr = 5.23; h=7 mils; t=2.8 mils w=10 mils; Calculate the characteristic impedance Z0 of the line.

**Solution:**



**Example:** A microstrip line is made of a copper conductor 0.254 mm (10 mils) wide on a G-10 fiberglass-epoxy board 0.20 mm (8 mils) in height. The relative dielectric constant εr of the board material is 4.8, measured at 25 GHz. The microstrip line 0.035-mm (1.4 mils) thick is to be used for 10 GHz. Assume conductivity of copper as 5.96×107 .Determine the

1. Characteristic impedance Z0 of the microstrip line
2. Surface resistivity Rs of the copper conductor
3. Conductor attenuation constant αc
4. Quality factors Qc

**Solution:**

Given εr=4.8; w=0.254 mm; h=0.20 mm; t=0.035 mm

1. Characteristic impedance Z0 of the microstrip line



1. Surface resistivity Rs of the copper conductor

 Ω/square. (11.196)

Where σ is the conductivity of the dielectric substrate of the copper board in ; the typical value is 5.96×107.



1. Conductor attenuation constant αc

 dB/cm

1. 

**Summary**

* A waveguide is a conducting tube through which the energy is transmitted, in the form of electromagnetic waves.
* The waveguide is a sharp high-pass filter with slightly increasing attenuation with respect to frequency.
* The waveguide operating modes are TE or TM modes (cannot support a TEM mode).
* A transverse electric (TE) modes have Ez = 0 and Hz  0.
* A transverse magnetic (TM) modes have Hz = 0 and Ez  0.
* The cut-off frequency is the operating frequency below which attenuation occurs and above which propagation takes place.
* The dominant mode is the lowest mode that can be used for the frequency of interest.
* Whenever two or more modes have the same cut-off frequency, they are called degenerate modes.
* Cut-off frequency of TEM mode is zero.
* The *wave impedance* of a waveguide is defined as the ratio of the strength of electric field in one direction and the magnetic field along the other transverse direction at a certain point in the waveguide.

* The phase velocity **(Vp)** of a waveguide is defined as the rate at which the wave changes its phase in terms of the guide wavelength per unit time.
* Cut off wave length of Rectangular wave guide is

fc=

* Cavity resonators are metallic enclosures that confine the electromagnetic energy within it.
* A circular resonant cavity is a circular waveguide with both its ends closed.
* The resonant frequency for a rectangular cavity is given by,



* The Quality factor (Q) is a measure of selectivity of the resonant circuit.



* Microstrip line consists of a conductor strip and ground plane. The electromagnetic wave propagates in quasi TEM mode.
* The major advantage of microstrip over stripline is that all active components can be mounted on top of the board. The disadvantages are that when high isolation is required such as in a filter or switch, some external shielding may have to be considered.
* The characteristic impedance of a microstrip line is



* There are 3 types of losses in a microstrip line.

1. Dielectric loss
2. Ohmic loss
3. Radiation loss

**Review Questions**

1. Why TEM mode is not possible for rectangular waveguides?
2. A rectangular waveguide has the following values l=2.54 cm, b= 1.27 cm waveguide thickness = .0127. Calculate the cut off frequency?
3. Explain the wave impedance of a rectangular wave – guide and derive the expression for the wave impedance of TE,TM, and TEM mode?
4. An air filled rectangular waveguide has dimensions of a=6cm and b=4 cm. the signal frequency is 3 GHz. Compute the following for the TE10, TE01, TE11, and TM11 modes:
5. Cut-off frequency
6. Wavelength in the guide
7. Phase constnat and phase velocity in the waveguide.
8. Group velocity and wave impedance in the guide.
9. A rectangular waveguide is filled by dielectric material of εr=9 and has dimensions of 7X3.5 cm. it operates in the dominant TE10 mode.
10. Determine the cut-off frequency.
11. Find the phase velocity in the guide at a frequency of 2 GHz.
12. Find the guided wavvelength λg at the same frequency.
13. A rectangular waveguide has dimensions *a* = 6 cm and *b* = 4 cm.
14. Over what range of frequencies will the guide operate single mode?

b) Over what frequency range will the guide support *both TE*10 and *TE*01 modes and no

others?

1. An air-filled rectangular waveguide is to be constructed for single-mode operation at 15 GHz. Specify the guide dimensions, *a* and *b*, such that the design frequency is 10/while being 10% lower than the cutoff frequency for the next higher-order mode.
2. Derive the expression for cut off frequency, phase constant and phase velocity of

wave in a circular wave guide?

1. An air-filled circular waveguide of 2 cm inside radius is operated in the TE01 mode.
2. Compute the cut-off frequency.
3. If the guide is to be filled with a dielectric material of εr=2.25, to what value must its radius be changed in order to maintain the cut-off frequency at its original value.
4. An air-filled circular waveguide has a radius of 1.5 cm and is to carry energy at a frequency of 10 GHz. Find all TE and TM modes for which transmission is possible.
5. A circular waveguide has a cut-off frequency of 9 GHz in dominant mode.
6. Find the inside diameter of the guide if it is air-filled.
7. Determine the inside diameter of the guide if the guide is dielectric filled. The ralative dielectric constant is εr=4.
8. What is meant by cavity resonator? Derive the expression for the resonant

frequency of the rectangular cavity resonator?

1. Derive the expression for the resonant frequency of the circular cavity resonator?
2. A rectangular cavity resonator has dimensions of a=5cm, b=2cm, and d=15 cm. compute:
3. The resonant frequency of the dominant mode for an air-filled cavity.
4. The resonant frequency of the dominant mode for a dielectric filled cavity of εr=2.56.
5. A circular cavity has a radius of 3.5 cm and 6 cm long. The line is dielectric –filled with εr=2.25.
6. Determine the resonant frequency of the cavity for TEM001.
7. Calculate the quality Q of the cavity.
8. Derive the expression for the Characteristic impedance of Microstrip Lines.
9. What are the various losses in a microstrip line? Explain.
10. A microstrip line is constructed of a copper conductor and nylon phenolic board. The relative dielctric constant of the board material is 4.19, measured at 25 GHz, and its thickness is 0.4836mm (19 mils). The line width is 0.635 mm (25 mils), and the line thickness is 0.071 mm (2.8 mils). Calculate the
11. Characteristic impedance Z0 of the microstrip line.
12. Dielectric attenuation constant αd
13. Surface skin resistivity Rs of the copper conductor at 25 GHz.
14. Conductor attenuation constant αc.
15. List out the different wave impedances for positive and negative directions?
16. What is Hybrid mode?

**Multiple Choice Questions**

1. The waves in a waveguide

a). travel along the border walls of the waveguide

b).are reflected from side walls but do not travel along them

c). travel through the dielectric without touching the walls

d). travel along the all the four walls

2. Wave guides can carry

a) TE mode b) TM mode c) Mixed mode d) All

3. The cut-off frequency of a waveguide depends on

a). dimensions of the waveguide b). the dielectric property of the medium in the waveguide c). wave mode d) all

4. In RWG, the mode subscripts m and n indicate

a) no of half wave patterns b) No. of full wave patterns c) no of the zeros of the field d) None

5. Wave impedance of wave-guide in TE mode can be

a)  b) 

c) both d) None

6. The dominant TE mode in rectangular wave guide is

a) TE01 b) TE11 c) TE20 d) TE10

7. Cut off wave length of Rectangular wave guide is

a)  b) 

c) both d) none

8. In RWG, for dominant mode, the cut off- wave length is

a) 2a b) 2b c) a d) None

9. An air filled rectangular waveguide has dimensions of 6 X 4cm.

Its cut-off frequency for TE10 mode is

a) 2.5GHz b) 25GHz c) 25MHz d) 5GHz

10. In hollow rectangular waveguides

a). the phase velocity is greater than the group velocity

b). The phase velocity is greater than the velocity of light in free space

c) both d) none

11. Dominant mode in circular wave guide is

a) TE10 b) TE11 c) TE01 d) TE12

12. Non existent modes in circular wave guides are

a) TE10 b) TE00 c) both d) None

13. Degenerate modes in circular wave-guides are

a) TE01 & TM11 b) TE22 & TM22 c) Both d) None

14. In cylindrical waveguide *Z TE* is

a) b) c)  d) 

15. Theoretically no. of modes that can exist in cylindrical waveguides

a) Zero b) One c) 2 d) Infinite

16. Guide wave length of cylindrical wave guide is



c) both d) None

17. Dominant modes in regular cavity resonators can be

a) TE111 b) TE110 c) Either d) None

18. Attenuation constant due to conductor loss is



19. The primary mode in a rectangular resonant cavity

a) TE111 b) TE101 c) TE100 d) TE001

20. Resonant frequency of a rectangular cavity resonator is



21. The transmission system using two ground planes is

a) Microstrip b) Rectangular waveguide

c) Circular waveguide d) Strip line

22. A disadvantage of strip line over microstrip is its

a) Easier integration with semiconductor devices

b) Lesser tendency to radiate

c) Higher isolation between circuits d) Higher ‘Q’

**Answers:** 1. **a** 2. **d** 3.**d** 4.**a** 5.**a** 6.**d** 7.**a** 8.**a** 9.**a** 10.**c** 11.**b** 12.**b** 13.**a** 14.**b** 15.**d** 16.**c** 17.**a** 18.**a** 19.**b** 20.a 21.d 22.a

**KEY EQUATIONS**

1. Relationship between phase and group velocity

C=

1. Relationship between frequency and free-space wavelength

C=F

1. Phase velocity as a function of wavelength

Vp=

1. Wavelength in a wave-guide

1. Cut-off wavelength for TE10 mode
2. **Cut-off frequency**
3. **Cut-off wavelength**



**REFERENCES**

1. SOUTHWORTH, G.E., Principles and Applications of Waveguide Transmission. D. Van Nostrand Company, Princeton, N.J., 1950.
2. SAAD, T., and R.C. HANSEN, Microwave Engineer’s Handbook, Vol.1. Artech House, Dedham, Mass., 1971.
3. WELCH J.D., and H.J.PRATT, Losses in microstrip transmission systems for integrated microwave circuits, NEREM Rec., 1966.
4. COHN, S., Characteristic impedance of the shielded- strip transmission line. IRE Trans. Of Microwave Theory and Techniques, MTT-2, No. 7, 52, July 1954.

[] O. HeavisideE, lectromagnetiTch eory,v ol. l, 1893.R eprintedb y Dover,N ew York, 1950.

[2] LordRayleigh,"OnthePassageofElectricWavesThroughTubes,"Philos.Mag.,vol.43,pp.125-132,

1897.R eprintedinC ollectedP apersC, ambridgeU niv.P ress,1 903.

[3] K. S.P ackard, "TheO rigino f WaveguidesA: Caseo f MultipleR ediscoveryI]E' EE TransM. icrowave

Theorya ndTechniquesv,o l. MTT-32,p p.961-969,S eptembe1r 984.

[4] R. M. Barett, "MicrowaveP rintedC ircuits-An HistoricalP erspectiveI,E" EE Trans.M icrowave

Theorya ndTechniquevso,l .M TT-32,p p.983-990S, eptembe1r9 84.

l5l D. D. Grieg and H. F. Englemann," Microstrip-A New TransmissionT echniquef or the KilomegacycleR

ange,"P roc. IRE,v ol. 40, pp.1644-1650D, ecembe1r 952.

[6] H. Howe,J r.,S triplineC ircuitD esign,A rtechH ouseD, edhamM, ass.,1 974.

t7l I. J.B ahla ndR . Garg,' A Designer'sG uidet o StriplineC ircuitsl'M icrowavesJ,a nuary1 978p, p.9 0-

96.

[8] I. J. Bahl and D. K. Trivedi,'A Designer'sG uide to Microstrip Linel' Microwaves,May1 977,

pp.174-182.

t9l K. C. Gupta,R . Garg,a ndI . J. Bahl,M icrostripL inesa nd Slotlines,A rlechH ouse,D edhamM, ass.,

t979.